15.4 Videos Guide

15.4a

- Mass of a planar lamina with shape D and density $\rho(x,y)$
  - $m = \iint_D \rho(x,y)\, dA$

- Total charge over an area with shape D and charge density $\sigma(x,y)$
  - $Q = \iint_D \sigma(x,y)\, dA$

- Moments
  - About the x-axis: $M_x = \iint_D y\, \rho(x,y)\, dA$
  - About the y-axis: $M_y = \iint_D x\, \rho(x,y)\, dA$

- Center of mass
  - $(\bar{x}, \bar{y}) = \left(\frac{M_y}{m}, \frac{M_x}{m}\right)$

- Moments of inertia (second moments)
  - About the x-axis: $I_x = \iint_D y^2\, \rho(x,y)\, dA$
  - About the y-axis: $I_y = \iint_D x^2\, \rho(x,y)\, dA$
  - About the origin: $I_0 = \iint_D (x^2 + y^2)\, \rho(x,y)\, dA = I_x + I_y$

- Radii of gyration
  - With respect to the x-axis: $\overline{\overline{y^2}} = \frac{I_x}{m}$
  - With respect to the y-axis: $\overline{\overline{x^2}} = \frac{I_y}{m}$
  - $(\bar{\bar{x}}, \bar{\bar{y}})$ is the rotational analog to the center of mass

15.4b

- Probability
  - $P\big((X,Y) \in D\big) = \iint_D f(x,y)\, dA$, where f is the joint density function of X and Y
  - Properties of a joint density function: $f(x,y) \geq 0$ and $\iint_{\mathbb{R}^2} f(x,y)\, dA = 1$

- Expected values for f a joint density function of random variables X and Y
  - x-mean: $\mu_1 = \iint_{\mathbb{R}^2} x\, f(x,y)\, dA$
  - y-mean: $\mu_2 = \iint_{\mathbb{R}^2} y\, f(x,y)\, dA$